

Standard Cosmology Delayed

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Abstract

The introduction of a delay in the Friedmann equation of cosmological evolution is shown to result in the very early universe undergoing the necessary accelerated expansion in the usual radiation (or matter) dominated phase. Occurring even without a violation of the strong energy condition, this expansion slows down naturally to go over to the decelerated phase, namely the standard Hubble expansion. This may obviate the need for a scalar field driven inflationary epoch.

1 Introduction

The Standard Model of Cosmology, based on assumptions of large scale statistical homogeneity and isotropy, is an elegantly simple and powerful theory. Supplemented with the Standard Model of Particle Physics—both of which are continuum field theories based on local interactions—it explains the evolution of our expanding universe almost up to the point of the initial big bang singularity. However, despite this success, there are issues that remain unresolved—indeed, for various reasons it seems unlikely that these can be the fundamental theories.

As our understanding of local quantum field theories has deepened, it has become increasingly clear that any such theory is only an effective description valid at an appropriate scale. The existence of Planck units as scales of length and time (deduced purely on dimensional grounds) seems to point to a limit to which one may hope to push this formalism. String theory (from which local field theories emerge in the low energy limit) is an alternative formalism that incorporates non-local interactions at Planck scale [1]. Similarly, loop quantum gravity or spin foam too have an inherent non-locality associated with them [2,3]. In a relativistically invariant theory, nonlocality also implies interactions smeared in time and delayed reaction, in particular.

Delayed reaction in dynamical systems occupy a place of central importance in many areas of science. In particular, many biological systems are governed by delay differential equations (DDE) [4]. Examples are models of population growth [5], where they first made their appearance. A characteristic feature of DDE is the way the derivative at any instant of time depends on the function at earlier times. This translates to solutions radically different from those obtained in the zero-delay case. To wit, a first order linear DDE with real coefficients may admit oscillatory solutions. Delayed dynamics in the decay of unstable branes of string theory have been studied [6] and compared to biological systems [7].

In this article, we regard cosmological evolution as a dynamical system [8,9]. We show that the introduction of a delay in the Friedmann equation ameliorates some of the shortcomings of the standard cosmological model. For a rather general class of initial data in, say, the radiation dominated epoch, the early universe is found to undergo a brief phase of accelerated expansion. Further, this slows down naturally to a decelerated expansion and asymptotes to standard FRW evolution. Thus, a delayed reaction within standard cosmology seems to obviate the need for an inflationary epoch driven by a scalar field as demanded in the standard paradigm and, simultaneously, solves the associated graceful exit problem. What is more, the initial accelerated expansion is obtained without a violation the strong energy condition.

Admittedly, our model is phenomenological in its spirit. We shall not attempt to obtain the delayed dynamical equation from first principles. We recognize that this important issue needs to be addressed—for an approach see [10]. And while a phenomenological model of non-local gravity was shown to give accelerated expansion [11], models of this

type are demonstrated to be equivalent to multi-scalar-tensor theories [12]. Non-local effects motivated by string theory have previously been studied in the cosmological context, see *e.g.*, [13, 14].

2 The Friedmann Equation

Let us briefly recall a few essential facts of the standard FRW cosmology, described by the scale factor $a(t)$ that determines physical distances at time t . The dynamics of the universe at large scales is governed by the Friedmann equation

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{1}{3}\rho(t), \quad (1)$$

where $\rho(t)$ denotes the total energy density and following standard practice, we have used natural units, namely $8\pi G = c = 1$. Homogeneity and isotropy at large scales dictate that a and ρ depend only on time. Assuming that the universe expands adiabatically, the first law of thermodynamics gives

$$a^3 \Delta\rho + (\rho + p) \Delta(a^3) = 0. \quad (2)$$

Combined with Eq.(1), this results in an equation for the Hubble expansion parameter $H(t) \equiv \dot{a}/a$:

$$2\dot{H}(t) + 3H^2(t) = -p, \quad (3)$$

a relation that can be derived as well from the Einstein-Hilbert action of GTR. One may rewrite it as

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{1}{6} [\rho(t) + 3p(t)], \quad (4)$$

to see that, for an accelerated expansion to occur, $\rho(t) + 3p(t)$ must be negative, *i.e.*, the strong energy condition (SEC) needs to be violated. Clearly, this is not the case with usual matter or radiation. One way to achieve this is the inflationary paradigm [15–17], in which one considers a scalar field that rolls on a sufficiently flat potential. Such models are considered to be the most simple, and variants of this theme are widely used to model an accelerating universe.

We note that of the three equations (1), (2) and (3) (or equivalently, (4)), only two are independent. For example, the third follows when we take a time derivative of the first and use Eq.(2); and this is the approach we shall take in the following.

3 Delayed Friedmann Equation

We propose a modification of the central dynamical equation (1) by introducing a constant delay τ . Thus, we postulate that any change of the matter content in the universe has a

delayed effect on the evolution of the metric and hence described by the *delayed Friedmann equation*

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{1}{3}\rho(t - \tau). \quad (5)$$

This may modify the dynamics of the universe, and we shall argue that that has the potential to provide a solution to some of the problems of the Standard Cosmology.

The standard lore may, however, prejudice one against such a nonlocal deformation. Nevertheless, the fact is that any direct knowledge of dynamics goes down, at best, to time scales of $\mathcal{O}(10^{-27}\text{s})$. Thus, for τ much smaller than this, say, comparable to Planck time $t_{\text{Pl}} \sim \mathcal{O}(10^{-43}\text{s})$, changes such as that in Eq.(5) are entirely consistent with observations. Moreover, the microscopic theories [1–3] presently in vogue incorporate nonlocality in an essential way. Besides, gravity may well be an emergent phenomenon [18, 19] (see also [20, 21]), in which case the equations related to gravity are not bound to be local.

It should be noted, though, that while we have listed possible quantum (gravity) effects as the source of this delay, the rest of our analysis is purely classical. This, indeed, is in the spirit of effective theories.¹ And, as we shall show later, the numerical value of τ could be commensurate with energy scales far lower than M_{Pl} (*i.e.*, τ larger than t_{Pl}), thereby allowing us to neglect quantum gravity effects, at least in the first approximation. As the discerning reader would recognize, this approximation is intrinsic to all current theories of inflation.

The dynamics of the universe, in this scheme, is thus governed by Eqs.(2) and (5), which, in turn, imply

$$2\dot{H}(t) + 3H^2(t) = \rho(t - \tau) - \frac{H(t - \tau)}{H(t)}(\rho(t - \tau) + p(t - \tau)), \quad (6)$$

in place of Eq.(3). Let us reiterate that the proposed delayed dynamics is *ad hoc* and not derived from a ‘fundamental theory’. It might be argued that modifications may be made in various other ways. While this is certainly true, it turns out that many of the theoretically desirable changes lead to qualitatively similar behaviour. Hence, rather than discussing each alternative separately, we restrict ourselves to the one proposed above. We shall comment briefly on the other possibilities at the end.

Before we proceed to explore the consequences of the delay, let us emphasise that the smallness of τ will ensure that there is virtually no change in the late time evolution of the universe, or in the dynamics of heavenly bodies that evolve at macroscopic scales. In other words, there would be no discernible consequences of this modification on astrophysical scales.

¹Analogous is the case of conventional inflationary theories wherein the inflaton potential is related to supergravity or even string theoretic constructions.

4 Delayed Dynamics & Early Accelerated Expansion

In this section, we shall find a solution to the set of equations (2), (5) and (6) and discuss its properties. For the sake of simplicity, let us assume an equation of state of the form $p = w\rho$ (where w is a constant) for matter or radiation that permeates the universe at the earliest epoch. The value of w is $1/3, 0$ and 1 , respectively, for radiation, non-relativistic dust and stiff fluid. While it is true that the early universe had several different components of matter, the one with the largest w would have dominated dynamics at early epochs. In any case, the inclusion of multiple fluids do not materially affect our analysis.

The equation of state, together with the first law of thermodynamics relates $\rho = \rho_0 a^{-3(1+w)}$, where ρ_0 is an arbitrary constant of integration. Substituting this in Eq.(5), we have

$$\frac{d}{dt} \ln a(t) = \sqrt{\frac{\rho_0}{3}} [a(t - \tau)]^{-3(1+w)/2}. \quad (1)$$

We solve this using the method of steps [4] starting with the ‘initial condition’ $a(0 \leq t < \tau) = f(t)$ where $f(t)$ is a given function. For definiteness, let us consider $f(t) = t^\alpha$ with a constant α . (Note that, as long as $\alpha \leq 1$, the universe is actually decelerating in this epoch, with $\alpha < 0$ implying a collapse. Such a situation may develop for a variety of reasons, including quantum gravity effects or a prior (pre ‘Big Bang’) history of the universe—it is not derived from within our model.) It now amounts to solving an ODE in subsequent intervals of size τ . Of particular interest is the solution in the first interval:

$$a(t) = \tau^\alpha \exp \left(\sqrt{\frac{\rho_0}{3}} \frac{(t - \tau)^{1 - \frac{3}{2}(1+w)\alpha}}{1 - \frac{3}{2}(1+w)\alpha} \right), \text{ for } \tau \leq t < 2\tau. \quad (2)$$

It is evident from the above that an accelerated expansion or *inflation* is possible for a wide choice of α even with normal radiation or matter. For later times, the solution has to be obtained numerically. We display a few cases in Fig.1.

The following features of ‘delayed FRW dynamics’ seem worth pointing out.

- A phase of fast growth exists for a wide range of α . In fact, for $\alpha \leq 0$ (an initially static or even contracting phase), the universe expands exponentially or faster. A fast growth can also occur for $\alpha > 0$ as long as $(1 + w)\alpha < 2/3$. (Many features actually depend on this combination of α and w .) Beyond this value, the growth of the scale factor is decelerated, as in standard cosmology.
- Throughout the phase of accelerated expansion, the strong energy condition holds good, in direct contrast to received wisdom, and, unlike all models of accelerated growth known so far. Indeed, the usual condition for acceleration, namely $w < -1/3$ is now replaced by

$$\frac{H(t - \tau)}{H(t)} < \frac{2}{3(1 + w)}. \quad (3)$$

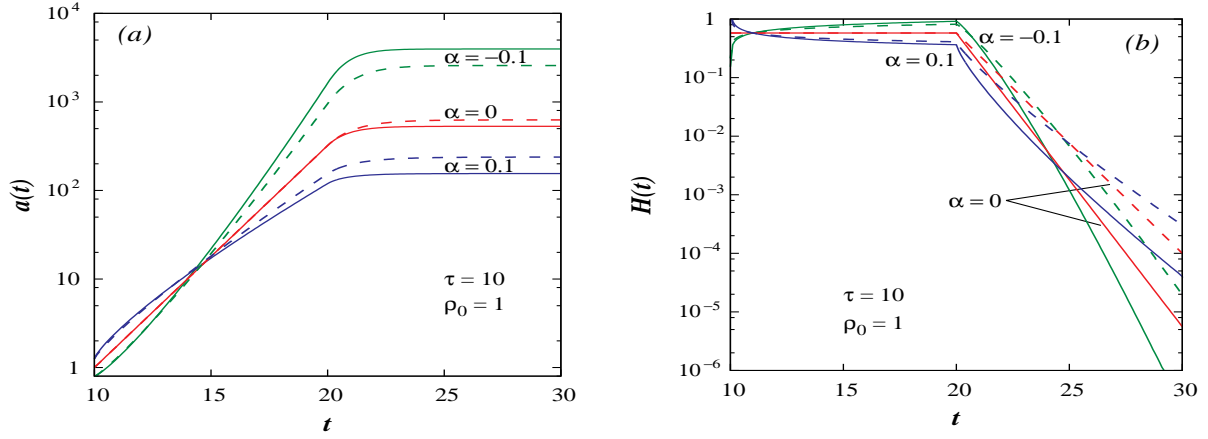


Figure 1: (a) The scale factor and (b) the Hubble parameter in the first two intervals. The solid and dashed line correspond to $w = 1/3$ and 0, respectively.

The requirement of $H(t - \tau)/H(t)$ being less than $1/2$ for radiation (or $2/3$ for nonrelativistic dust), is easy to satisfy for a large class of initial conditions. This, in essence, is the most important and interesting result to emerge out of the proposed delayed dynamics.

- The rate of growth is determined by the initial matter density ρ_0 and grows with it. Again, this would seem counterintuitive, for a larger energy density, instead of slowing down the expansion, actually increases it.
- The end of inflation is denoted by the onset of deceleration. In the present case, there is a subtlety. While, for $(1 + w)\alpha \leq 0$, the universe has an accelerated expansion during $\tau \leq t < 2\tau$, for $0 < (1 + w)\alpha < 2/3$, on the other hand, the universe is decelerating initially, but quickly passes onto an accelerating phase, with the onset of acceleration being progressively delayed for larger values. The initial decelerating phase, however, would leave virtually no signal in the sky.
- The duration of the ‘inflationary’ growth phase is of the same order as the delay τ , as the universe quickly settles down to a phase of decelerated growth after $t \gtrsim 2\tau$ (see Fig.1(b)). In most conventional models of inflation, ‘graceful exit’ is often a problem and a mechanism needs to be introduced to ensure that the accelerated expansion stops. Remarkably, the introduction of a delay in Eq.(5) not only introduces an inflationary phase, but also serves to bring the universe out of it. Naively, one would have expected that the exit would occur only for $t \gg \tau$, as, at such late times, the delay would be immaterial and the system would essentially revert to the normal power law expansion. Rather the exit is *precocious* in the present system. Fortunately, however, this does not cost us in terms of the number of e-foldings.

The exit from inflation is accompanied by an abrupt change in $\ddot{a}(t)$ (see Fig.1(b)). Such discontinuities (one such also occurs at $t = \tau$) in higher derivatives are endemic to DDEs with generic initial conditions. In the present context, this could potentially affect primordial density perturbations and consequent signatures in the background microwave spectrum as also the production of superheavy dark matter [22]. Further discussion of this, however, is beyond the scope of this work.

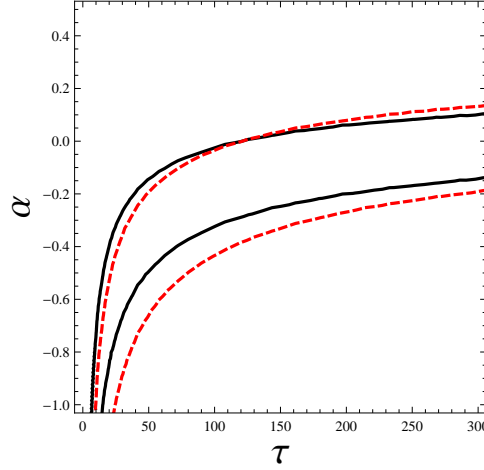


Figure 2: The contours for the number of e-folds in the α - τ plane. The solid and dashed line correspond to $w = 1/3$ and 0 , respectively. In each case, the upper curve is for 70 e-folds while the lower one is for 700 e-folds.

It is well known that for inflation to solve the problems of standard cosmology, we need at least about 65 e-folds (*i.e.*, growth in $a(t)$ by a factor of e^{65}) by the end of inflation. In Fig.2, we display constant-inflation contours in the τ - α plane for different values of w . Note that the dependence on w is small. In effect, the bulk of inflation occurs in the first epoch ($\tau \leq t \leq 2\tau$) where the dynamics is governed by the product $(1+w)\alpha$. It is only in the next phase, where the expansion slows down considerably and the ‘inflation’ ends that the separate dependence on w appears. This was to be expected in view of Fig.1. What is, perhaps, more interesting is that the delay required becomes smaller as α becomes more negative. In other words, the faster the universe was collapsing, the faster it rebounds back; since a nonzero τ implies that crunching of matter reacts on the space-time fabric only after a delay, it stands to reason that a larger compression needs to be bottled up only for a shorter duration before it reacts violently.

It is also quite apparent that, to achieve a phenomenologically acceptable amount of inflation starting with a static or slowly evolving ‘initial condition’ (*i.e.*, a small $|\alpha|$), one would require $\tau \sim \mathcal{O}(10^2\text{--}10^3)t_{\text{Pl}}$. One might interpret this as being a restatement of the requirement of the (usual) inflaton scale being $\sim M_{\text{GUT}}$. The analogy, however, is not exact. Rather, the amount of inflation can be approximated to be given by a

certain function $F_{\text{in}}(\tau, (1+w)\alpha)$ with only subsidiary dependence on other variables. The functional form of F_{in} does depend on the exact initial condition $a(0 \leq t < \tau)$, but, as we have demonstrated above, inflation occurs for a very wide class of initial conditions. It is worthwhile to point out at this stage that, contrary to popular wisdom, it is not necessary that inflation must occur at M_{GUT} or thereabouts. For example, as shown in Ref. [23], the invocation of a Planck-size proper-length cut-off automatically regulates the size of the density perturbations to acceptable limits. While the cut-off in Ref. [23] may have been introduced in an ad-hoc manner, its existence would be natural in theories that would lead to delays, such as ours.

5 Endnote

In this paper, we have initiated the study of an alternative mechanism for an accelerated expansion in the early universe. It is not driven by a scalar field, but rather achieved by modifying the Friedmann equations by the introduction of a delay, in a phenomenological fashion. We are aware that many questions have remained unanswered and our first effort in this direction is far from complete. While the answers to these would evidently require major effort, we shall touch upon some of these issues presently.

- *On the completeness of the theory:*

Let us reiterate that although non-locality abounds in quantum theories of gravity [1–3, 6, 10, 13, 14] and we have been motivated by its ubiquity, ours is only a phenomenological approach. Ideally, one should have a complete theory of gravity to study all the ramifications associated with the idea of inflation. It is needless to say that a naive modification of the Einstein equations cannot be the answer. The ideas propounded in Ref. [10] demonstrates that delayed dynamics in a covariant framework requires much subtlety. Meanwhile, we may consider an analogous dynamical equation describing the decay of an unstable brane [6, 24]. It is conceivable that once such branes are coupled to gravity, similar effects, in the presence of an appropriate background, would lead to delayed equations involving gravity. Admittedly, the complete dynamical system would be a much more complicated one than that we have considered. Thus, we only report on a preliminary study intended to stimulate the search for such field configurations.

- *On the uniqueness of the delay mechanism:*

It might rightly be claimed that the delayed dynamics we consider is not unique. Let us comment briefly on other possible ways of introducing delay in the dynamics. For example, if, in addition to (5), there is an identical delay in the RHS of (4) (in which ρ and p are taken to be related by the equation of state), the evolution of the scale factor is exactly the same as in the standard FRW universe. However, the

evolution of matter would differ. On the other hand, suppose we work with Eqs.(5) and (3) (which is *not* delayed), the time derivative of ρ turns out to depend on a and ρ at a time in the future, leading to an acausal behaviour.

It is of course possible to introduce a delay in many other ways, however, in some cases, the properties of the solution remain qualitatively same. For instance, Eqs.(1) and $\dot{\rho}(t) + 3(1+w)H(t-\tau)\rho(t) = 0$, a variant of Eq.(2), gives exactly the same evolution. If instead, one considers the variant $\dot{\rho}(t) + 3(1+w)H(t)\rho(t-\tau) = 0$, the qualitative behaviour of the evolution of the scale factor remains the same.

In a sense, therefore, we have introduced a *minimal* modification in which the delay occurs in only one equation, namely the Friedmann equation, which describes the effect of matter on the expansion of the universe. Eq.(2) is the statement of energy conservation; and since that is applicable in a wider context, it has not been modified.

- *On the stability of the system:*

Higher derivative theories are usually known to suffer from an inherent instability, known as the Ostrogradskian instability, which was extended to infinite number of higher derivatives in, *e.g.*, Refs. [26, 27]. It might be argued, with a delay being equivalent to the existence of an arbitrarily higher order derivatives, that the theory would necessarily be unstable. One should, however, be cautious to come to this conclusion without a detailed analysis. For one, the nonlocality due to the delay by τ , being equivalent to $e^{-\tau\partial_t}$, is through an entire function of the derivative (with respect to time) [26]. Moreover, such systems have been studied in the literature, with particular emphasis on the cosmological context in, *e.g.*, Refs. [7, 13, 25] (and references therein) and seem to exhibit a reasonable dynamical behaviour. It is, therefore, not unreasonable to expect that once the delayed Friedmann equation (perhaps in a form modified from the one we use) is obtained from a fundamental Lagrangian upon the inclusion of quantum corrections, the corresponding dynamical system would, generically, represent a stable system.

- *On the dependence on the initial data:*

Modifying finite order differential equations to delayed differential equations has necessitated the imposition of initial conditions over a *finite continuous segment*. It might be argued that this represents the introduction of an infinite number of new degrees of freedom. While this criticism is valid *per se*, we emphasise that the existence of an accelerating phase is not tied to a particular form of the initial data $f(t)$. We have checked numerically that the accelerated expansion is quite generic and occurs for a very wide choice of initial data. It is only that a monomial or a single exponential form for $f(t)$ permits a simple closed analytical form for the scale factor $a(t)$, and we have chosen to illustrate our arguments with the former. For more complicated forms for $f(t)$, the entire solution has to be obtained numerically.

We do not have *a priori* arguments in favour of any particular choice for $f(t)$, preferring to demonstrate the imprint of the delay on the early universe with a simple class of initial conditions. These could be a result of either a series of quantum fluctuations or the result of some cataclysmic *pre-Big Bang*² events (the latter possibility appears naturally in many theories of quantum gravity, including, but not limited to string theory). Indeed, as long as the *pre-Big Bang* universe was not expanding as fast as the corresponding un-delayed Friedmann equation would have wanted it to, a phase of accelerated expansion phase would necessarily occur in the delayed version.

Finally, to summarise, we have demonstrated the possibility that a delay introduced in the Friedmann equation could naturally lead to an exponential (or faster) growth phase in the very early universe. The existence of such a phase requires neither the existence of a scalar field (inflaton) with a flat potential nor even a violation of the strong energy condition. While our formulation is obviously a phenomenological one, it should essentially be considered a proof of principle motivated by the existing theories of quantum gravity. However, a more detailed construction based on a microscopic theory should be sought.

Not only does the universe inflate, it also asymptotes to FRW cosmology, thereby eliminating the need for an exit mechanism. The required delay is small (a few hundred Planck times, at most) and natural in the context of nonlocalities inherent in quantum gravity. It is also consistent with all observations. Work on subsequent reheating and generation of primordial fluctuations are in progress and will be reported elsewhere. Also of interest is the potential that a significant fraction of the primordial energy density could have existed in the form of magnetic fields (thereby offering a possible seed for the intergalactic magnetic field observed today), a scenario that is difficult to accommodate in canonical inflationary scenarios [28]. A rich tapestry of many such physical consequences may be expected.

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²Note that, in our context, it is $t = \tau$ that defines the beginning of time as we know it.

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